

# A Generalized Hybrid Approach to Controlling Emissions

*Svante Mandell*<sup>†</sup>

November 2004

## *Abstract*

This paper examines the optimal instrument choice to control emissions under uncertainty. A hybrid regulation mechanism is developed that contains cap-and-trade, emissions taxes and so-called safety valves as special cases. This makes it possible to examine optimal policy choice and the resulting efficiency losses for each instrument. It is shown that the hybrid regulation mechanism in efficiency terms is weakly superior to the other instruments. The model is also used to study optimal response to non-optimal policy implementations.

*JEL classification*; Q28, Q58, H23

*Keywords*; Emissions tax, Emissions trading, Safety valve, Ranking, Uncertainty

---

<sup>†</sup> Financial support from the Swedish Energy Administration is gratefully acknowledged. The paper has benefited from suggestions by Peter Bohm and Björn Carlén. Comments are welcome to Svante.Mandell@ne.su.se

## 1. Introduction

Human activities often result in harmful substances being emitted into air or water. Such externality problems are rarely solved by the market itself, but often call for governmental regulations. There are many regulation instruments discussed in the literature and/or used in practice. The present paper restricts its attention to a few of them, in particular emissions taxes and cap-and-trade. These instruments have the virtue of being cost effective, as they both equate the marginal abatement costs throughout the regulated economy.

In a deterministic setting, emissions taxes and cap-and-trade can be designed to yield identical and efficient outcomes. However, if abatement costs are uncertain this is no longer possible. Weitzman (1974) formally showed that a price instrument, *i.e.*, an emissions tax, yields a lower expected efficiency loss than cap-and-trade if the marginal abatement cost (*MAC*) function is steeper than the marginal abatement benefit (*MAB*) function and *vice versa*. The logic behind this result is that, given a relatively steep *MAC*, it may be very expensive to achieve a given level of abatement, as would be required if cap-and-trade were used. However, given a relatively steep *MAB*, the environmental costs may be very high if an emissions tax were used.

Robert and Spence (1976) introduced a hybrid regulation policy that may be described as cap-and-trade with an upper and a lower bound on the permit price. A simplified version of that mechanism is known as ‘safety valve’, see *e.g.* Jacoby and Ellerman (2004). A safety valve works as a cap-and-trade regulation up to a price level that, if reached, triggers a reversion to a tax regime. That is, if the price per emission permit would be too high, due to the realization of the *MAC*, more permits will be issued on the market at a pre-determined price. Pizer (2002) studies efficiency consequences of adopting a safety valve mechanism using numerical simulations, capturing many of the results derived analytically in the present study.

In the present paper a regulation mechanism is developed, referred to as a ‘generalized hybrid mechanism’. It is similar to the safety valve as it includes a cap and, as a trigger, a price level. However, it differs from the safety valve approach as we allow for a price schedule with positive slope, *i.e.* the price may increase in the number of permits issued.

There are two benefits with this model. First, it will be shown that the generalized hybrid mechanism generally is better, from an efficiency point of view, than the best of the other three instruments. Second, by restricting the generalized hybrid mechanism in different ways

we may mimic the other instruments and, hence, the model allows us to study all four instruments in one common framework.

There are other studies that combine price and quantity regulations. Weitzman (1978) introduces a regulation that works as a price instrument with a built-in penalty variable that will operate if a firm deviates from a pre-set quantity target. If the penalty variable is large enough this scheme will operate as a standard cap-and-trade regulation – any deviation from the target is then prohibitively costly – and if the variable is zero it will operate as an emissions tax. Kennedy *et. al.* (2003) study optimal pollution pricing schemes. It is similar to the present study in that it allows for the price per unit of emissions to increase in the emissions made. A crucial difference between the present study and these two is that the present study, as the aforementioned Robert and Spence (1976), allows for trade in permits, which equates marginal abatement costs *ex post*.

The remaining paper is structured as follows. Section 2 introduces the model and derives the optimal levels of the policy variables. In section 3 the generalized hybrid mechanism is restricted so that it behaves as the other three instruments respectively. The optimal levels of the policy variables for each of these instruments are then derived. In section 4 the results derived in section 2 and 3 are used to rank the four instruments with respect to efficiency. Section 5 looks at situations where some policy variable(s) are not optimally set. The question is when and how the policy maker can respond in such situations through calibrations of other variables. Section 6 concludes.

## 2. The model

In line with most of the earlier literature on this subject, we restrict attention to quadratic abatement cost functions and abatement benefit functions, such that the *MAC* and the *MAB* both are linear. Arithmetically, we have

$$MAC(e, \varepsilon) = K - Le + \varepsilon \tag{1}$$

$$MAB(e, \delta) = f + ge + \delta \tag{2}$$

where  $e$  is aggregate emissions and  $K, L, f$  and  $g$  are non-negative constants. For a positive emissions level to exist in equilibrium we must have  $K > f$  and that, at least,  $L$  or  $g$  is strictly positive.  $\varepsilon$  and  $\delta$  are independent random variables. The general intuition from the model

remains for all distributions of  $\varepsilon$  and  $\delta$  that are symmetric around zero. For simplicity we assume  $\varepsilon \sim U(-a, a)$  and  $\delta \sim U(-b, b)$ .

The model presumes the following timing: At date 1, the policy maker decides on, declares and commits to a regulation policy package. At date 2, permits are allocated to market participants. At date 3, the values of the random variables are realized. At date 4, trade takes place under the restrictions imposed by the policy package such that a market clearing price is established.

The policy package is to be decided upon before the values of  $\varepsilon$  and  $\delta$  are realized. It is assumed that the policy maker's objective is to implement a policy package that minimises the expected dead weight loss,  $E\{DWL\}$ . Three policy variables are at the policy maker's disposal, all of which can take on non-negative values. First, the emissions cap, denoted  $q$ . Second, a price level, over which additional permits will be issued to the market. This level is denoted  $s$ , and referred to as the 'Price Schedule Trigger' or, for short, PS-trigger. Third, the slope of the price schedule, denoted  $\beta$ . That is, if the market clearing price under the cap,  $q$ , reaches  $s$ , additional permits will be issued according to a pre-determined price schedule,  $T(s, \beta, q; e)$ . We restrict attention to linear price schedules. Hence  $T$  may be written as

$$T(s, \beta, q; e) = s + \beta(e - q) \quad (3)$$

To illustrate, say that the policy package consists of  $q_1$ ,  $s_1$  and  $\beta_1$ . This implies that  $q_1$  permits are allocated to market participants. If  $\varepsilon$  is realized such that the market clearing price under the cap  $q_1$  is below the PS-trigger,  $s_1$ , the price schedule never operates. However, if  $\varepsilon$  turns out high, such that the market clearing price under  $q_1$  would exceed  $s_1$ , the policy maker will issue more permits according to the price schedule,  $T_1(s_1, \beta_1, q_1; e)$ . The market clearing price will then be such that the *MAC* equals the price schedule,  $T_1$ .

The efficient aggregate emissions level,  $e^*$ , solves  $MAC(e^*, \varepsilon) = MAB(e^*, \delta)$ . Hence

$$e^* = \frac{K - f - \delta + \varepsilon}{L + g} \quad (4)$$

The price schedule will operate if the clearing price under the cap would become too high, *i.e.* if  $MAC(q, \varepsilon) > s$ . From this, a threshold value, denoted  $\varepsilon^T$ , can be calculated

$$\varepsilon^T = s - (K - Lq) \quad (5)$$

For reasons that will become apparent, we must have  $-a \leq \varepsilon^T \leq a$ . Let us refer to this as the ‘interior-requirement’ for future reference. When  $\varepsilon \leq \varepsilon^T$  the policy package works as a normal cap-and-trade regulation, the number of permits on the market will be  $q$  and the market clearing price  $MAC(q, \varepsilon)$ . If, on the other hand,  $\varepsilon > \varepsilon^T$  the number of permits on the market, denoted  $e^T$ , is given by equating  $MAC(e, \varepsilon)$  with  $T(s, \beta, q; e)$ . This yields

$$e^T = \frac{K - s + \beta q + \varepsilon}{L + \beta} \quad (6)$$

The market clearing price is then given by  $MAC(e^T, \varepsilon)$ .

When  $\varepsilon \leq \varepsilon^T$  the realized  $DWL$  is given by

$$\int_{e^*}^q (MAB(e, \delta) - MAC(e, \varepsilon)) de = \int_{\frac{K-f-\delta+\varepsilon}{L+g}}^q (f + ge + \delta - (K - Le + \varepsilon)) de \quad (7)$$

which yields

$$\frac{(f + gq + \delta - (K - Lq + \varepsilon))^2}{2(L + g)} \quad (8)$$

When, on the other hand,  $\varepsilon > \varepsilon^T$  the realized  $DWL$  is given by

$$\int_{e^*}^{e^T} (MAB(e, \delta) - MAC(e, \varepsilon)) de = \int_{\frac{K-f-\delta+\varepsilon}{L+g}}^{\frac{K-s+\beta q+\varepsilon}{L+\beta}} (f + ge + \delta - (K - Le + \varepsilon)) de \quad (9)$$

this yields<sup>1</sup>

$$\frac{(K(g - \beta) + L(f - s + \beta q + \delta) + \beta(f + \delta - \varepsilon) + g(-s + \beta q + \varepsilon))^2}{2(L + g)(L + \beta)^2} \quad (10)$$

---

<sup>1</sup> For intuition, note that the exact same results as in (8) and (10) can be reached by elementary trigonometry. (8) is the area of a triangle with height  $q - e^*$  and base  $MAC(q, \varepsilon) - MAB(q, \delta)$ . The same goes for (10) but with height  $e^T - e^*$  and base  $MAC(e^T, \varepsilon) - MAB(e^T, \delta)$ .

Using the threshold value,  $\varepsilon^T$ , from (5) and the  $DWL$ :s from (8) and (10) we can express the expected  $DWL$  following from a policy package as

$$E\{DWL\} = \frac{1}{4ab} \int_{-a}^{\varepsilon^T} \int_{-b}^b (DWL \text{ from (8)}) d\delta d\varepsilon + \frac{1}{4ab} \int_{\varepsilon^T}^a \int_{-b}^b (DWL \text{ from (10)}) d\delta d\varepsilon \quad (11)$$

Thus, the policy maker's objective is to minimise (11) by choosing an appropriate policy package consisting of  $q$ ,  $s$  and  $\beta$ . From inserting (5), (8) and (10) in (11) we have<sup>2</sup>

$$\begin{aligned} \min_{q,s,\beta} [E\{DWL\}] = & \min_{q,s,\beta} \left[ \frac{1}{4ab} \int_{-a}^{s-(K-Lq)} \int_{-b}^b \left( \frac{(f + gq + \delta - (K - Lq + \varepsilon))^2}{2(L + g)} \right) d\delta d\varepsilon + \right. \\ & \left. \frac{1}{4ab} \int_{s-(K-Lq)}^a \int_{-b}^b \left( \frac{(K(g - \beta) + L(f - s + \beta q + \delta) + \beta(f + \delta - \varepsilon) + g(-s + \beta q + \varepsilon))^2}{2(L + g)(L + \beta)^2} \right) d\delta d\varepsilon \right] \end{aligned} \quad (12)$$

The relevant (minimising) first order conditions yield

$$\begin{aligned} q^* = & \frac{1}{L^2(L^2 + 3g\beta + 2L(g + \beta))} (KL(L + g)(L + 2\beta) - \\ & L(L^2 f + Lgs + 2gs\beta + L\beta(f + s)) - a(L + g)(L^2 + 2L\beta + 2\beta^2) + (L + \beta) \\ & \sqrt{L^2(Lf - ag + Kg - (L + g)s)^2 - 4aL(L + 2g)(Lf + Kg - (L + g)s)\beta + 4a^2(L + g)^2\beta^2}) \end{aligned} \quad (13)$$

$$s^* = \frac{(a + K)(g - \beta) + 2\beta(f + gq) + L(2f + q(g + \beta))}{2L + g + \beta} \quad (14)$$

$$\beta^* = \frac{L(3f + gq - 3s) + 2g(a + K - s)}{2(a + K - Lq) - 3(f + gq) + s} \quad (15)$$

By appropriate substitutions we can solve for each policy variable, which yields

$$q^* = \frac{K - f - a}{L + g} \quad (16)$$

$$s^* = f + gq^* = \frac{Lf + Kg - ag}{L + g} \quad (17)$$

---

<sup>2</sup> Looking at the integration limits we see why the interior-requirement is needed. If the requirement is not fulfilled we force the model to integrate over a wider range than  $(-a, a)$ .

$$\beta^* = g \quad (18)$$

There is an intuitive explanation for these equations. First, note that we can rewrite (16) as

$$K - Lq^* - a = f + gq^* \quad (19)$$

which shows that the optimal cap,  $q^*$ , is given by the emissions level where the *MAC*-function with the lowest possible location, *i.e.* with  $\varepsilon = -a$ , equals the expected *MAB*. This insight together with expression (17) – note that the mid term of (17) is identical to the right hand side of (19) – shows that the optimal PS-trigger,  $s^*$ , is such that the price schedule always, *i.e.* for all realizations of  $\varepsilon$ , will be binding. Also, note that inserting  $s^*$  and  $\beta^*$  in the expression for the price schedule yields

$$T^*(e, s^*, \beta^*, q) = s^* + \beta^*(e - q) = f + gq + ge - gq = MAB(e | \delta = 0) \quad (20)$$

Hence, the optimal policy package entails setting a price schedule that exactly mimics the expected *MAB*-function and a cap such that this price schedule always binds. Thus, it is similar to a non-linear tax scheme where the tax, *ex post*, is set equal to the marginal damage from aggregate emissions, see, *e.g.*, Kaplow and Shavell (2002) and references therein. However, in a dynamic setting the two systems are likely to differ. Under a non-linear tax scheme, firms have limited information of what the tax will be when they emit, since they cannot observe emissions from other firms. Under the generalized hybrid mechanism, trade is allowed, which is likely to provide at least partial information on other firms' demand of permits.

### 3. Three special cases

The model presented in the previous section is general in the sense that it contains both cap-and-trade and emissions taxes, as well as the standard safety valve as special cases.

Consequently, by restricting the model we can solve for optimal policy variables given the choice of regulation approach. This will now be shown.

#### *Cap-and-trade*

By forcing the PS-trigger to be high enough the price schedule will never, *i.e.*, not under any realization of  $\varepsilon$ , enter into force and hence the regulation will work as a pure cap-and-trade

regulation. Denote this restricting level of the PS-trigger by  $s_{CnT}$ . To capture the effect of a cap-and-trade regulation, the following must be true:

$$s_{CnT} \geq K - Lq + a \quad (21)$$

The right hand side of (21) is the highest possible *MAC* at  $e = q$ . From the interior-requirement we will use the  $s_{CnT}$  at which (21) is binding<sup>3</sup>. By inserting (21), and letting it bind, in (13), and solving for  $q$  we reach the following optimal cap

$$q_{CnT}^* = \frac{K - f}{L + g} \quad (22)$$

which is such that it equates the expected *MAC* with the expected *MAB*. Note that when (21) holds, the price schedule never enters into force and consequently we do not have to specify  $\beta$ .

#### *Emissions tax*

An emissions tax is such that for all emissions levels there is a uniform price. To capture this with the present model we need to 1) force the cap to be such that, for all realizations of  $\varepsilon$ , the price schedule binds and 2) force the price schedule to be flat, *i.e.*, set  $\beta$  to zero.

The first point is achieved by introducing a cap,  $q_p$ , that fulfils the following restriction, which is the opposite of the restriction given in (21);

$$s \leq K - Lq_p - a \quad (23)$$

This states that, at  $e = q_p$ , the lowest possible *MAC* must be at least equal to  $s$ . Hence, for all realizations of  $\varepsilon$  the price schedule will be binding. Letting (23) bind and solving for  $q_p$  yields

$$q_p = \frac{K - s - a}{L} \quad (24)$$

---

<sup>3</sup> Consider any  $s_{CnT} > K - Lq + a$ . Intuitively, this should not alter the results since the price schedule in any case will not enter into force. Mathematically, however, it does, since the integration limits would be outside the valid range of  $(-a, a)$ , see (12). Hence, for technical reasons we must use the  $s_{CnT}$  where (21) binds.



Inserting this into (14) yields

$$s = \frac{Lf + Kg}{L + g} - \frac{a\beta}{L + \beta} \quad (25)$$

The second point, forcing a uniform price by setting  $\beta$  to zero, trivially yields

$$s_p^* = \frac{Lf + Kg}{L + g} \quad (26)$$

Which is, thus, the optimal tax. Note that the optimal tax is equal to the market clearing price that follows from equating the expected  $MAC$  with the expected  $MAB$ . Consequently, it is true that  $s_p^* = E\{MAC(q_{CnT}^*)\}$ .

Two things in the discussion above might seem to fit badly with an emissions tax and deserve some comments. First, trade is still allowed within the model. However, under the imposed restrictions there will be only one seller, namely the regulator, which has an infinite amount of permits to sell at a pre-determined fixed price. Second, the inclusion of a cap in general, and a non-zero cap in particular, in a tax regime might seem inappropriate, but note that the role of the cap in the model is to trigger the price schedule – and it is set to do so for every possible realization. Hence, under the restrictions mentioned, the model captures the effects of an emissions tax.

### *Safety valve*

The model captures the elements of a safety valve simply by forcing the price schedule to be flat, *i.e.*, setting  $\beta$  to zero, without imposing any further restrictions on  $q$  and  $s$ . To find the optimal cap-level,  $q_{SV}^*$ , in the safety valve, first insert (14) into (13) and then set  $\beta$  to zero.

This yields

$$q_{SV}^* = \frac{K - f}{L + g} - \frac{aL}{(L + g)^2} \quad (27)$$

From (27) it is seen that the optimal cap under a safety valve is the optimal cap under cap-and-trade *minus* an expression that, when  $L > 0$ , is non-negative and increases in the uncertainty around the  $MAC$ . That is,  $q_{SV}^* \leq q_{CnT}^*$ . By rewriting (27) in the same manner as (19) we get:

$$K - Lq_{sv}^* - \frac{aL}{L+g} = f + gq_{sv}^* \quad (28)$$

From (28) it is seen that for all  $g > 0$  and  $L > 0$ , there are realizations of  $\varepsilon$  such that the cap will be binding.<sup>4</sup> For  $g = 0$ , *i.e.* when  $MAB(e/\delta)$  is constant, the optimal safety valve comprises a cap level identical to the optimal cap under the price schedule – which, from (18), intuitively should be the case. This implies that for all  $g > 0$  we have that  $q_{sv}^* > q^*$  and that  $q_{sv}^* = q^*$  if  $g = 0$ .

The optimal safety valve level (*i.e.*, the optimal PS-trigger),  $s_{sv}^*$ , is found in a similar manner; insert (13) in (14) and, in the resulting expression, set  $\beta = 0$ . This yields

$$s_{sv}^* = \frac{Lf + Kg}{L+g} + \frac{ag^2}{(L+g)^2} \quad (29)$$

Comparing (29) with (26) shows that the optimal safety valve is equal to the optimal tax *plus* an expression that, when  $g > 0$ , is non-negative and increases in the uncertainty around the *MAC*. Hence, when  $g > 0$  the optimal safety valve level,  $s_{sv}^*$ , is strictly larger than the optimal emissions tax,  $s_p^*$ . From (17) we have an expression for the optimal PS-trigger under the generalized hybrid mechanism, which for  $g > 0$  is strictly less than the optimal emissions tax. Hence,  $s^* < s_p^* < s_{sv}^*$  for all  $g > 0$ . If  $g = 0$  and  $L > 0$  then  $s^* = s_p^* = s_{sv}^* = f$ . If  $L = 0$ , *i.e.*  $MAB(e/\varepsilon)$  is constant, while  $g > 0$  we get  $s^* (= K - a) < s_p^* (= K) < s_{sv}^* (= K + a)$ .

## 4. Ranking of instruments

Thus far we have derived the optimal levels of the policy variables under four different regulation instruments: the generalized hybrid mechanism and, by imposing appropriate restrictions on the model, cap-and-trade, emissions tax and safety valve. We are now in a position where we may, for each instrument, calculate the resulting  $E\{DWL\}$  given the

---

<sup>4</sup> In (28) the optimal cap is such that it equates the expected *MAB*-function, the right hand side of (28), with the *MAC*-function given by the left hand side. Since this *MAC*-function, for  $L > 0$ , lies strictly above the lowest possible one, see the left hand side of (19), there are (low) realizations of  $\varepsilon$  such that the safety valve will work as cap-and-trade.

optimal policy variables. As we regard an instrument that generates smaller expected deadweight losses as better, this enables us to rank the different regulation instruments.

A general expression for the  $E\{DWL\}$  is given by (12). For each instrument respectively, inserting into (12) the expressions for the optimal policy variables and the valid restrictions discussed in the preceding section yields the expected deadweight losses. Table 1 presents the results from such an operation. Column 1 contains the expression for  $E\{DWL\}$ . Column 2 – 4 contains three special cases. In column 2, we set  $L = g$ , *i.e.* the slope of the *MAC* equals the slope of the *MAB*. In column 3, we set  $L = 0$ , which imposes a constant *MAC*. In column 4,  $g$  is set to zero, *i.e.* the *MAB* is constant. Column 5 reports the loss function for each instrument relative to cap-and-trade calculated as  $E\{DWL\}$  for the instrument in question minus  $E\{DWL\}$  under cap-and-trade. The variance of  $\varepsilon$  and  $\delta$  is denoted  $\sigma_\varepsilon^2$  and  $\sigma_\delta^2$  respectively.

**Table 1** Expected deadweight losses,  $E\{DWL\}$ , for each instrument respectively and loss function relative to cap-and-trade. <sup>\*)</sup>  $\lim_{L \rightarrow 0} E\{DWL_{\text{Price}}\} \rightarrow \infty$

	$E\{DWL\}$				$\text{Loss Function}$
	Column 1	Column 2 $L=g$	Column 3 $L=0$	Column 4 $G=0$	Column 5
Cap-and-Trade	$\frac{\sigma_\varepsilon^2 + \sigma_\delta^2}{2(L+g)}$	$\frac{\sigma_\varepsilon^2 + \sigma_\delta^2}{4g}$	$\frac{\sigma_\varepsilon^2 + \sigma_\delta^2}{2g}$	$\frac{\sigma_\varepsilon^2 + \sigma_\delta^2}{2L}$	-
Emissions tax	$\frac{\frac{g^2}{L^2} \sigma_\varepsilon^2 + \sigma_\delta^2}{2(L+g)}$	$\frac{\sigma_\varepsilon^2 + \sigma_\delta^2}{4g}$	Not defined <sup>*)</sup>	$\frac{\sigma_\delta^2}{2L}$	$\left(\frac{g^2}{L^2} - 1\right) \frac{\sigma_\varepsilon^2}{2(L+g)}$
Safety Valve	$\frac{\frac{g^2}{(L+g)^2} \sigma_\varepsilon^2 + \sigma_\delta^2}{2(L+g)}$	$\frac{\frac{1}{4} \sigma_\varepsilon^2 + \sigma_\delta^2}{4g}$	$\frac{\sigma_\varepsilon^2 + \sigma_\delta^2}{2g}$	$\frac{\sigma_\delta^2}{2L}$	$\left(\frac{g^2}{(L+g)^2} - 1\right) \frac{\sigma_\varepsilon^2}{2(L+g)}$
General Hybrid Mech.	$\frac{\sigma_\delta^2}{2(L+g)}$	$\frac{\sigma_\delta^2}{4g}$	$\frac{\sigma_\delta^2}{2g}$	$\frac{\sigma_\delta^2}{2L}$	$-\frac{\sigma_\varepsilon^2}{2(L+g)}$

Let us first compare the emissions tax with cap-and-trade in the light of the analysis conducted in Weitzman (1974). There it is shown that when  $L > g$ , *i.e.* the *MAC* is steeper than the *MAB*, an emissions tax is better than cap-and-trade, *i.e.*, a tax yields lower  $E\{DWL\}$ . For  $L < g$  the opposite applies and for  $L = g$  the two instruments yield the same  $E\{DWL\}$ . The first case is illustrated in column 4 in table 1, where cap-and-trade yields an  $E\{DWL\}$  that is  $\sigma_\varepsilon^2/2L$  larger than that under an emissions tax. Hence the latter is preferable. The second case is illustrated in column 3 where the  $E\{DWL\}$  from the emissions tax tends to infinity as

$L \rightarrow 0$ . The third case is seen in column 2 where both instruments result in identical expected efficiency losses. The loss function in column 5 is negative if  $g < L$  and positive if  $g > L$ . A negative value on the loss function implies that an emission tax is preferred over cap-and-trade.

Turning to the  $E\{DWL\}$  following from a safety valve regime we see from column 2-4 that a safety valve in all these cases performs at least as well as cap-and-trade and an emissions tax. From the loss function it is seen that, for all  $g > 0$  and  $L > 0$ , an optimal safety valve is strictly better, *i.e.* yields strictly lower  $E\{DWL\}$ , than both an optimal cap-and-trade and an optimal emissions tax. The reason is that the safety valve reduces the impact from the uncertainty around the  $MAC$ . That is,  $\sigma_\varepsilon^2$  has less impact on  $E\{DWL\}$  in the safety valve expression than in cap-and-trade and the emissions tax. This is neatly illustrated in column 2 ( $L = g$ ) where, under a safety valve,  $\sigma_\varepsilon^2$  enters the expression as  $\sigma_\varepsilon^2/4$  while it enters as  $\sigma_\varepsilon^2$  in the other cases.

Finally, let us turn to the generalized hybrid mechanism. First note that for the special cases, reported in column 2-4, the generalized hybrid mechanism performs at least as well as the other instruments. Furthermore, looking at the loss function we see that for all  $g > 0$  and  $L > 0$ , the generalized hybrid mechanism strictly outperforms all other instruments. In the light of our previous discussion, this is hardly surprising. We have shown that the other instruments are special cases of the generalized hybrid mechanism and, hence, the optimal generalized hybrid mechanism can never be worse than the other instruments, as it can always mimic them<sup>5</sup>. As noted, when  $g > 0$  and  $L > 0$  the safety valve outperforms an emissions tax and cap-and-trade from limiting the impact of the  $MAC$ -uncertainty,  $\sigma_\varepsilon^2$ . The same is done by the generalized hybrid mechanism but to a larger extent, since the impact of  $\sigma_\varepsilon^2$  here is reduced to zero.

## 5. Remedies to non-optimal policies

Thus far it has been shown that when  $L$  and  $g$  are both positive the best instrument, from an efficiency point of view, is a generalized hybrid mechanism designed such that the price

---

<sup>5</sup> Similarly, the safety valve weakly outperforms an emissions tax and cap-and-trade since it gives the regulator more degrees of freedom by being able to decide on both  $s$  and  $q$ .

schedule always binds and perfectly mimics the expected  $MAB$ . One interesting question is then: given that for political or other reasons some policy variable is not set according to equations (16), (17) and (18), is it possible to decrease the impact on  $E\{DWL\}$  by calibrating the other policy variables and how should this be done? Here we restrict attention to the case with a non-optimal cap but similar exercises could be carried out for the other variables as well. Say that the implemented cap is given by

$$q_{imp} = q^* + \Delta = \frac{K - f - a}{L + g} + \Delta \quad (30)$$

*i.e.* the optimal cap under a generalized hybrid mechanism plus an arbitrary constant,  $\Delta$ , which – in order to fulfill the interior requirement – lies in the interval of zero to  $2a / (L + g)$ . Inserting (30) in (14) and (15) and solving for the optimal PS-trigger,  $s_{imp}^*$ , and price schedule slope,  $\beta_{imp}^*$ , yield

$$s_{imp}^* = \frac{Lf + Kg - ag}{L + g} + g\Delta \quad (31)$$

$$\beta_{imp}^* = g \quad (32)$$

From (32) it is seen that  $\Delta$  has no impact on the optimal slope. However, from (31), it has crucial impact on the optimal PS-trigger. Equation (31) states that the larger the deviation from the optimal cap,  $\Delta$ , the larger is the optimal PS-trigger. That is, when  $g > 0$  we may remedy the negative impact on  $E\{DWL\}$  from a non-optimal cap by calibrating the PS-trigger such that the less stringent the cap is, the more stringent the PS-trigger should be. Closer examination of (31) reveals how the compensation works. The first term of (31) equals  $s^*$ , *i.e.* the price at the lowest possible intersection of  $MAC$  and  $E\{MAB\}$ . The second term is simply the slope of the  $MAB$  times  $\Delta$ . Hence, the optimal compensation in PS-trigger, following the implementation of a non-optimal cap, is such that once the price-schedule is in force it will follow the expected  $MAB$  function. However, this does not fully compensate for the non-optimal cap. This is seen by inserting (30), (31) and (32) in (12), which yields an expression for  $E\{DWL\}$  under  $q_{imp}$

$$E\{DWL \mid q_{imp}\} = \frac{b^2}{6(L + g)} + \frac{(L + g)^2 \Delta^3}{12a} \quad (33)$$

The first term equals  $E\{DWL\}$  under an optimized generalized hybrid mechanism. The second term disappears when  $q_{imp} = q^*$ , *i.e.* when  $\Delta = 0$ , and is positive for all  $q_{imp} > q^*$ . That is, the further from the optimal cap the implemented cap is, the higher the expected deadweight loss. Note that  $\Delta$  appears in (33) raised to the power of three, *i.e.* there is an exponentially increasing effect on  $E\{DWL\}$  from implementing a non-optimal cap further from the optimal one. Clearly, if the PS-trigger is not compensated the  $E\{DWL\}$  will be even larger. The  $E\{DWL\}$  from a non-optimal cap and an uncompensated PS-trigger equal to  $s^*$  amounts to

$$E\{DWL \mid q_{imp}, s_{imp} = s^*\} = \frac{b^2}{6(L+g)} + \frac{(L+g)^2 \Delta^3}{12a} \frac{6ag^2 + L^2(L+3g)\Delta}{(L+g)^3 \Delta} \quad (34)$$

which is (33) with the second term multiplied by a factor larger than 1<sup>6</sup>. Hence,  $E\{DWL\}$  is greater if the PS-trigger is not compensated for the non-optimal cap.

Let us also briefly look at the above setting with an additional restriction of  $\beta = 0$ , *i.e.* a safety valve with non-optimal cap. Still using the  $q_{imp}$  from (30) and forcing  $\beta$  to zero yields an optimal PS-trigger:

$$s_{SVimp}^* = \frac{(L+g)(a+K) + L(a-K+f)}{L+g} - \frac{4aL}{2L+g} + \frac{Lg\Delta}{2L+g} \quad (35)$$

First note that if  $\Delta$  is such that  $q_{imp}$  equals  $q_{SV}^*$ , *i.e.*  $\Delta = a g / (L+g)^2$ , (35) simplifies to  $s_{SV}^*$ , which should be the case since this constitutes the optimal cap / PS-trigger combination under a safety valve. Second, note that differentiating (35) with respect to  $\Delta$  yields  $Lg / (2L+g)$ , which is strictly positive under  $L$  and  $g > 0$ . That is, the higher the non-optimal cap is the higher is the optimal responding PS-trigger, *i.e.* if  $q_{imp} > q_{SV}^*$  then  $s_{SVimp}^* > s_{SV}^*$  and *vice versa*. This seems to be analogous to the former case, where  $\beta$  need not be zero. However, the compensation scheme differs between the two cases. Intuitively, if the policy maker is unable to implement a positively sloping price schedule to capture a positively sloping *MAB* function

---

<sup>6</sup> At  $g = 0$  the factor equals 1. For  $g > 0$  the factor is greater than 1 both at  $\Delta = \Delta^{\min} = 0$  and  $\Delta = \Delta^{\max} = 2a / (L+g)$ . Differentiating with respect to  $\Delta$  yields a strictly negative term and, consequently, the factor cannot take on values less than 1 for any  $\Delta$  inside the interval.

the next best thing is to calibrate the PS-trigger upwards. That is, given a non-optimal cap, we would expect the PS-trigger under a safety valve to be larger than the one under a price schedule regulation, *i.e.* that  $s_{SVimp}^* > s_{imp}^*$ , when  $g > 0$ . That this indeed is the case is seen by subtracting  $s_{imp}^*$ , given by (31), from  $s_{SVimp}^*$ , given by (35). This yields

$$s_{SVimp}^* - s_{imp}^* = \frac{g(L+g)}{2L+g} \left( \frac{2a}{L+g} - \Delta \right) \quad (36)$$

which is strictly positive for all  $g > 0$  and  $\Delta$  fulfilling the interior requirement (except when  $\Delta$  takes on its highest possible value,  $2a / (L+g)$ , at which  $s_{SVimp}^* = s_{imp}^*$ ).

Thus far we have concentrated on emissions such that  $g > 0$ . For some emissions this is not necessarily the case. As an example, let us look at CO<sub>2</sub>, probably the major cause of climate change. CO<sub>2</sub> tends to stay in the atmosphere over long periods of time. As a consequence, the environmental damage it causes is not very sensitive to emissions made in one particular time period, but rather it depends on the cumulated concentration in the atmosphere<sup>7</sup>. From the viewpoint of any given point in time, this implies a *MAB* that is virtually horizontal, *i.e.* a  $g$  close to zero<sup>8</sup>. One of the special cases in the former section discussed the case with an approximation of  $g = 0$ . The conclusion was that a price schedule regulation, a safety valve or an emissions tax will then all be optimal and generate exactly the same  $E\{DWL\}$ . A cap-and-trade regulation will, however, result in a strictly larger  $E\{DWL\}$  and hence, from an efficiency point of view, is inferior to the other alternatives. That the price schedule regulation and the safety-valve generate identical  $E\{DWL\}$  as an emissions tax is not surprising, since the previous analysis shows us that, when  $g = 0$ , the two instruments perfectly mimic the emissions tax in optimum.

---

<sup>7</sup> Here we use CO<sub>2</sub> emissions mostly to illustrate our results. To better capture its nature a dynamic model, which is able to handle the interdependence of stock and flows of emissions, is needed. See *e.g.* Hoel and Karp (2001, 2002) for such models and further references.

<sup>8</sup> See Jacoby and Ellerman (2004) for a further discussion about this and whether a safety valve may be useful in a climate policy context.

The former analysis states that the optimal generalized hybrid mechanism contains a cap such that the price schedule always operates. Again, consider the situation where, *e.g.* for political reasons, a non-optimal cap is implemented. From the previous discussion, we know that this has no impact on the price schedule slope. When  $g = 0$ , however, it is seen from (31) that it does not influence the optimal PS-trigger either. Thus, when regulating emissions with constant *MAB*-functions it is the case that implementing a generalized hybrid mechanism with a cap such that the instrument differs from a pure emissions tax is sub-optimal and it is not possible to remedy the problem through compensations in the PS-trigger or the slope of the price-schedule.

## 6. Conclusions

In the present paper we have introduced and analyzed a generalized hybrid regulation mechanism, which allows for cap-and-trade up to a pre-determined trigger price. If the price under cap-and-trade tends to exceed the trigger, additional permits will be issued on the market in accordance with a pre-determined price schedule. The difference between this generalized hybrid mechanism and the standard safety valve is that the price schedule need not be flat. Instead the price may increase in the number of permits issued.

Two major conclusions may be drawn from the analysis. First, it has been shown that, barring the extreme case when the total abatement benefit function is linear, the price schedule regulation always results in lower expected deadweight losses compared to the other studied instruments, *i.e.*, cap-and-trade, emissions tax and safety valve. Under a linear total benefit function, *i.e.*, marginal abatement benefits are constant in emissions, the optimal versions of the generalized hybrid mechanism, the safety valve and an emissions tax are all equivalent. Hence, given that policy makers are concerned with efficiency and that the underlying assumptions are reasonably valid, the generalized hybrid mechanism is a strong policy instrument candidate.

Second, as the generalized hybrid mechanism contains the three other instruments as special cases, the model can easily be used to study all these regulation mechanisms inside one common framework. In the present paper we have, for example, studied the expected deadweight losses resulting from optimal use of each of the regulations respectively. From this it is concluded that the optimal safety valve is better than optimal cap-and-trade and optimal emissions tax as it lessens the impact from the uncertainty surrounding the abatement costs. The generalized hybrid mechanism regulation goes even further and eliminates this



impact entirely<sup>9</sup>. The model also allows us to study other aspects, *e.g.* how the optimal values of the policy variables relate to each other.

Additionally, the model may be used to examine situations where, *e.g.*, for political reasons, some of the policy variables are set at non-optimal values. In the paper we have addressed situations where the implemented cap differs from the optimal one. We show that this may cause the policy maker to respond by calibrating other policy variables in order to minimize the efficiency loss and how this should optimally be done. Generally, such calibrations improve the result.

Several interesting questions that may be subject for future research arise from the present analysis. Here, two will be mentioned. First, to keep the model tractable we have made some simplifying assumptions on the cost and benefit functions. Basically, we have used the same assumptions as applied in most of the preceding studies in this field. In particular, the attention is limited to quadratic functions, *i.e.* the marginal abatement cost function and the marginal abatement benefit function are assumed to be linear. It would be interesting to see how non-linear functions would influence the results derived in this paper – both in the price schedule case and under a safety valve. Furthermore, as the model is designed the uncertainty only affects the level of the marginal abatement cost and benefit functions. The implications of also introducing uncertainty about the slopes of these functions would clearly be of interest. Second, in the present paper the model is static. One interesting topic for future research is to study this problem in a dynamic setting. Presumably, this will have impact on the model, especially in the price schedule and the safety valve settings.

---

<sup>9</sup> This exact result naturally depends on the assumption used, in particular that the marginal abatement benefit function and the price schedule are both linear.

## References

- Hoel Michael and Karp Larry (2001) Taxes and quotas for a stock pollutant with multiplicative uncertainty, *Journal of Public Economics*, **82**, 91-114
- Hoel, Michael and Larry Karp (2002) Taxes versus quotas for a stock pollutant, *Resource and Energy Economics*, **24**, pp. 367-384
- Jacoby Henry D. and Ellerman A. Denny (2004) The Safety Valve and Climate Policy, *Energy Policy*, **32**, 481-491
- Kennedy Peter W., Laplante Benoit and Whittington Dale (2003) Increasing Marginal Pricing Schemes for Pollution Control Under Asymmetric Information, prepared for the 2003 Australasian Meetings of the Econometric Society
- Pizer William A. (2002) Combining price and quantity controls to mitigate global climate change, *Journal of Public Economics*, **85**, 409-434
- Roberts M. J. and Spence M. (1976) Effluent charges and licenses under uncertainty, *Journal of Public Economics*, **5**, 193-208
- Weitzman Martin L. (1974) Prices vs. Quantities, *Review of Economic Studies*, **41**, 477-491
- Weitzman Martin L. (1978) Optimal Rewards for Economic Regulation, *American Economic Review*, **68**:4, 683-691